Dyakonov Surface Waves in Photonic Metamaterials

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We show that suitable photonic metamaterial structures can support lossless surface waves of the form envisaged by Dyakonov. The idea we put forward is based on the birefringent properties of photonic crystals in the long-wavelength limit, and uses the metamaterial anisotropy of the structures to meet the resonance conditions at which Dyakonov surface waves exist. Implementation of this concept can lead to the first experimental observation of Dyakonov waves, and might open the door to the realization of widely tunable sensing devices based on the unique properties of such waves.

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Surface waves are a special type of waves that are confined at the very boundary between two different media [1,2]. They feature genuine physical phenomena as well as prospects for far-reaching applications. In particular, by their very nature, surface waves are unique tools to explore the properties of material interfaces. This includes not only intrinsic properties but also extrinsic effects, thus making surface waves ideal tools for sensing physical, chemical, and biological agents. For example, the widespread current applications of surface plasmons range from subwavelength microscopy, near-field optical tweezing, and enhanced light-matter interactions [3–6] to molecular chemistry, proteomics, and cancer research [7–9].

A unique type of surface waves was discovered theoretically by Dyakonov more than a decade ago [10]. Like plasmons polaritons, they exist at the surface of two different materials, and should feature similar excitation and detection properties. However, in contrast to plasmons, Dyakonov surface waves exist in transparent media; thus they are lossless. More importantly, they exist only under rare conditions at the interface between two media when at least one of them is anisotropic. In the simplest case of the interface between a uniaxial crystal and an isotropic medium, Dyakonov surface waves exist only for positive birefringence (when the extraordinary refractive index of the substrate, n_e, is larger than the ordinary refractive index, n_o), provided that the refractive index of the cover isotropic medium, n_c, verifies the inequality

$$n_e > n_c > n_o. \quad (1)$$

This is a condition not easy to meet in practice with natural materials, because the difference between the extraordinary and the ordinary refractive indices of most transparent materials is typically tiny; thus the band of allowed refractive indices n_c is very narrow. Analogous stringent requirements hold for the existence of Dyakonov waves in all the configurations that have been studied theoretically so far [11–14]; hence, Dyakonov surface waves have never been observed to date. In this Letter we expose that the required existence conditions can be met in suitable photonic structures operating at the long-wavelength limit. In addition, and crucial from the point of view of future applications, we show that, owing to the metamaterial properties of the composites, the condition (1) can be met over a wide range of refractive indices with high sensitivity. This suggests the application of the phenomenon to a new type of widely tunable sensing devices based on guided-to-leaky, or on-off, plasmonic-type resonance transitions.

Different types of modes confined at the edge of photonic crystal structures are known to exist and to feature rich physical phenomena and prospects of important applications [15–17]. However, in contrast to above-mentioned modes, Dyakonov waves are linked to the full anisotropic nature of the supporting surface. Suitable photonic crystal composites in the long-wavelength, or homogenization, limit are known to behave as effective birefringent crystals [18–22]; thus they might provide the anisotropy required for the formation of Dyakonov waves. In this work we explore such a possibility for the first time to our knowledge.

The first step of this program is the identification of structures that are uniaxial and that exhibit positive birefringence. One-dimensional photonic crystals are always uniaxial, with the optical axis orthogonal to the dielectric interfaces. Two-dimensional crystals can be both uniaxial or biaxial [22]. Three-dimensional photonic crystals can be designed to be isotropic, uniaxial, or biaxial. One finds that one-dimensional geometries cannot exhibit a positive birefringence. In periodic composites, the dielectric constant for the electric field parallel to the dielectric interfaces always is given by the weighted dielectric constant $\bar{\varepsilon}$ [20]. In one-dimensional photonic crystal structures, this electric field corresponds to the ordinary wave, and thus, the effective ordinary refractive index is given by $n_{eo}^2 = \bar{\varepsilon}$ (subscript e indicates "effective"). For a multilayer made of two media with refractive indices n_1 and n_2, and filling factor f, this quantity writes $\bar{\varepsilon} = n_1^2 f + n_2^2(1 - f)$. Similarly, the effective extraordinary refractive index is given by the weighted reciprocal dielectric constant $n_{ee}^2 = (\bar{\eta})^{-1}$ [15], which writes $\bar{\eta} = f/n_1^2 + (1 - f)/n_2^2$. It fol-

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lows from these expressions that $n_{ee} < n_{eo}$; thus the composite features a negative birefringence. Thus, here we address two-dimensional structures, with the general layout depicted in Fig. 1. To elucidate the effective refractive indices of the structure, we use the homogenization approach in the long-wavelength limit outlined in Ref. [22]. Homogenized photonic crystals whose unit cell has a third- or higher-order rotational axis behave as uniaxial crystals. The rotational axis parallel to the rods is the effective optical axis. Then, the effective extraordinary refractive index is given by $n_{ee}^2 = \bar{\varepsilon}$. For the ordinary refractive index of the homogenized structure, one finds

$$n_{eo}^2 = \left(\bar{\eta} - \frac{1}{2}\sum_{\vec{G},\vec{G}'\neq 0} \vec{G}\cdot\vec{G}'\eta(\vec{G})\eta(-\vec{G}')\mathbf{M}^{-1}(\vec{G},\vec{G}')\right)^{-1}, \tag{2}$$

where $\vec{G}$ is the reciprocal vector of the two-dimensional lattice, and $\mathbf{M}(\vec{G},\vec{G}') = \vec{G}\cdot\vec{G}'\eta(\vec{G}'-\vec{G})$. This quantity has to be evaluated numerically, taking into account the properties of the unit cells. One then finds that the photonic structures considered here always feature a positive birefringence. This property is guaranteed by the Weiner bounds [20] that give the maximum value of an effective dielectric constant as $\varepsilon_{\text{eff}} \leq \bar{\varepsilon}$. In Fig. 2 we plot the effective ordinary and extraordinary refractive indices of the structure as a function of the filling factor for composites with cylindrical and square holes, in silicon and in silica. The plot directly exposes the main advantage afforded by photonic composites in comparison to natural uniaxial crystals for the purpose in hand: n_{ee} can be made much larger that n_{eo}. Also it shows the possibility of engineering the anisotropic properties of the structure not only by changing the material but also by changing the unit cell geometry and filling factor.

Dyakonov surface waves are hybrid, with all the field components. The fields in the cover and substrate can be written as $\vec{E}_i(\vec{r},t) = \vec{E}_i(x)\exp[j(\vec{\kappa}\,\vec{r} - \omega t)]$. Here $\vec{\kappa}\,\vec{r} = k_0 N(y\sin\theta + z\cos\theta)$, where k_0 is the free-space wave number, N is the effective index of the guiding surface modes, and θ is the propagation angle with respect to the optical axis. The complex amplitude $\vec{E}_i$ must be evanescent in the x direction. The evanescent amplitude is a superposition of extraordinary and ordinary modes, namely, $\vec{E}_s(x) = \vec{E}_{os}\exp[k_0\gamma_{os}x] + \vec{E}_{es}\exp[k_0\gamma_{es}x]$, where $\gamma_{os} = \sqrt{N^2 - n_{eo}^2}$ and $\gamma_{es} = \sqrt{N^2[\sin^2\theta + (n_{ee}^2/n_{eo}^2)\cos^2\theta] - n_{ee}^2}$. Similarly, the field amplitude in the cover writes $\vec{E}_c(x) = \vec{E}_c\exp[-k_0\gamma_c x]$ with $\gamma_c = \sqrt{N^2 - n_c^2}$. Finally, the boundary conditions at the interface yield the eigenvalue equation for the existence of guided surface waves [10],

$$\tan^2\theta = \frac{\gamma_{os}(\gamma_c + \gamma_{os})(n_{eo}^2\gamma_c\gamma_{es} + n_c^2\gamma_{os}^2)}{n_{eo}^2(\gamma_c + \gamma_{es})(n_{eo}^2\gamma_c + n_c^2\gamma_{os})}, \tag{3}$$

an expression that relates the propagation angle, the cover refractive index, and the structural properties of the photonic crystal.

The central physical result dictated by (3) is that hybrid surface waves exist only within an interval of propagation angles θ, typically narrow, which in the physical setting addresses here is implicitly related to the properties of the photonic structure through the reciprocal vectors of the photonic lattice appearing in (2). Such an interval is given by the cutoff conditions for resonance with the continuous spectrum. The cutoff for radiation into the cover continuous spectrum, namely, $\gamma_c = 0$, gives the minimum allowed angle, $\theta_{\min}$, for Dyakonov waves to exist, while the maximum allowed angle, $\theta_{\max}$, is obtained for the cutoff for coupling into the substrate continuous spectrum of extraordinary polarized waves, given by $\gamma_{es} = 0$. The allowed interval $\Delta\theta = \theta_{\max} - \theta_{\min}$ is found to be approximately centered at the propagation angle where both $\gamma_{es} = 0$ and $\gamma_c = 0$, namely,

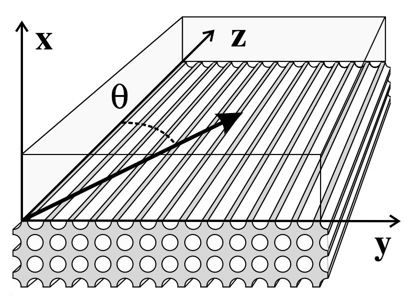

FIG. 1. Sketch of the particular structure analyzed here, consisting of a cover made of an isotropic material and a substrate photonic crystal. The lattice constant a of the 2D photonic crystal is much smaller than the excitation wavelength. The concept holds for different cell shapes and for general anisotropies.

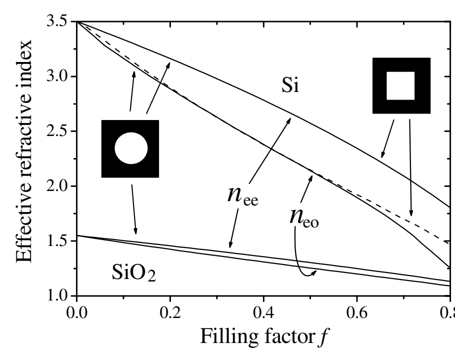

FIG. 2. Effective ordinary and extraordinary refractive indices of homogenized structures, calculated by solving numerically Eq. (2), as a function of the filling factor. The solid lines correspond to a unit cell of cylindrical holes ($n_1 = 1$) embedded in Si ($n_2 = 3.5$) and SiO_2 ($n_2 = 1.55$). Unit cells with different geometries but the same filling factor produce identical results for n_{ee}, while n_{eo} can change. This is shown by the dashed line, corresponding to a unit cell of square holes embedded in Si.

$$\theta_0 \approx \sin^{-1}\left[\frac{n_{ee}}{n_c}\sqrt{\frac{n_c^2 - n_{eo}^2}{n_{ee}^2 - n_{eo}^2}}\right], \quad (4)$$

an expression to be evaluated numerically in terms of the filling factor using (2). The parameters θ_0 and $\Delta\theta$ can be modified by varying the properties of the photonic crystal. This possibility is illustrated in Fig. 3. Note that $\Delta\theta$ can amount to a few degrees, while with most natural uniaxial crystals the allowed interval might be 2 orders of magnitude smaller [13].

Dyakonov surface modes excited on photonic metamaterials seem to be particularly well suited for versatile plasmons-polariton type sensing. This concept is based on the condition (4), which suggests the use of Dyakonov waves to detect the presence of extrinsic agents, like liquids or biological samples, by placing them on top of the metamaterial. The potential of this possibility is illustrated in Fig. 4, which shows how the central allowed propagation angle varies with the cover refractive index for five illustrative filling factors. The plot shows that by varying continuously the filling factor one can tune the central existence angle to match any desired cover refractive-index range.

The experimental implementation of the concept put forward in this Letter can be achieved in a photonic periodic structure where the homogenization approach holds. Such a regime is encountered when the slope of the wave dispersion relation is linear, e.g., for wavelengths below half the first photonic band edge, when the lattice constant is sufficiently smaller in comparison to the operating wavelength, i.e., $\lambda \gg a$. Meeting such a condition at optical wavelengths in the interval 1–5 μm requires lattice periodicities and cell sizes in the range 0.1–1 μm, while with GHz radiation the homogenization approach is expected to hold with lattice sizes of the order of the *mm*. On the other hand, the penetration depth $\Gamma_{ms} = \lambda_0/(2\pi\gamma_{ms})$, with $m = e, o$, of the evanescent tails associated with the Dyakonov wave inside the photonic structure has to be much larger than the cell size and the metamaterial periodicity, i.e., $\Gamma_{ms} \gg a$.

To elucidate whether such is the case and to examine the degree of confinement of the predicted surface waves, we comprehensively studied the evanescent decay rate of the Dyakonov waves supported by a variety of photonic metamaterials. We found that, when $\lambda \gg a$, the condition mentioned above always holds. An illustrative example is shown in Fig. 5(a), which displays the variation of the penetration depths Γ_{es} and Γ_c, as a function of the propagation angle of a typical surface wave, inside the interval of allowed propagation angles. Notice that even near the cutoff condition for resonance with the continuum spectrum in the cover, when the penetration depth of the wave inside the metamaterial is the smallest, Γ_{es} is of the order of the wavelength. Thus, as long as the condition $\lambda \gg a$ is fulfilled, the surface wave extends over many lattice cells, as required. Figure 5(b) further clarifies the point. The plot shows the minimum value of Γ_{es} that occurs at the cutoff for coupling into the cover continuous spectrum and that is given by $\Gamma_{es,\min} = \lambda_0/[2\pi\gamma_{es}(N = n_c; \theta = \theta_{\min})]$, as a function of the cover refractive index, for all the filling factors considered in Fig. 4. As is visible in the plot, the higher the cover refractive index, thus the lower the filling factor of the photonic structure, the more the evanescent wave extends into the structure, hence the more justified the homogenization approach we used in here to derive the existence of the Dyakonov waves.

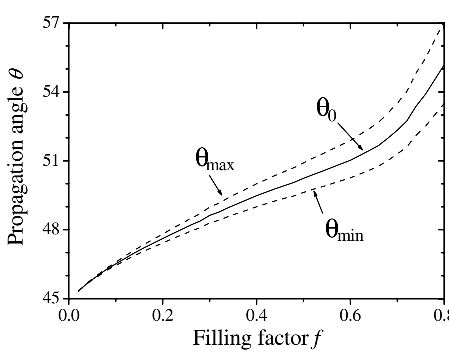

FIG. 3. Allowed band of propagation angles for the Dyakonov surface waves, as a function of the filling factor of a photonic structure made of cylindrical hole cells in silicon. The dashed lines stand for the maximum and minimum cutoff angles. The solid line is the central angle, given by (4). The refractive indices n_{ee} and n_{eo} are given by Fig. 2, and in this particular plot we set $n_c = (n_{ee} + n_{eo})/2$.

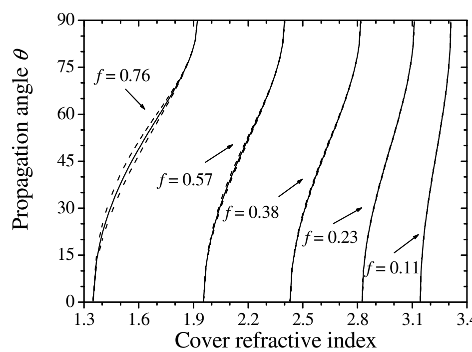

FIG. 4. Central allowed propagation angle versus cover refractive index for different filling factors. The data correspond to a photonic structure consisting of cylindrical rods in silicon, but qualitatively similar results are obtained for other geometries The ordinary and extraordinary effective indices for each curve are the following: $n_{ee} = 1.922$ and $n_{eo} = 1.350$ ($f = 0.76$); $n_{ee} = 2.380$ and $n_{eo} = 1.956$ ($f = 0.57$); $n_{ee} = 2.818$ and $n_{eo} = 2.43$ ($f = 0.38$); $n_{ee} = 3.111$ and $n_{eo} = 2.824$ ($f = 0.23$); $n_{ee} = 3.312$ and $n_{eo} = 3.144$ ($f = 0.11$). In all cases, the maximum refractive index for the cover corresponds to n_{ee} and the minimum to n_{eo}. The dashed lines, barely visible in most cases, display the values of $\theta_{\max}$ and $\theta_{\min}$.

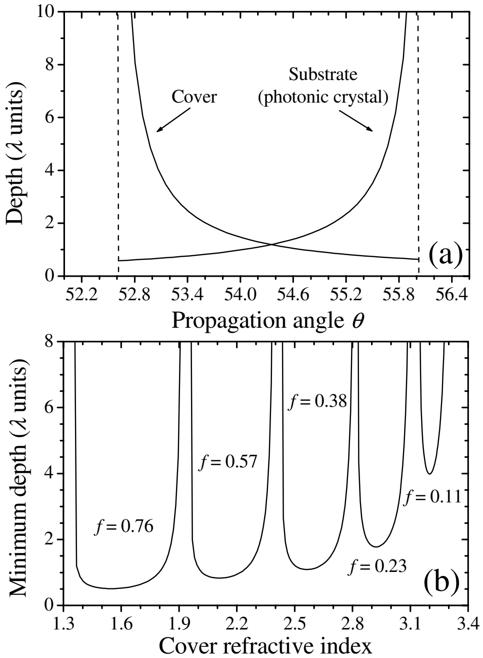

FIG. 5. (a) Evanescent wave penetration depth, Γ_{es}, Γ_c into a photonic crystal substrate and cover for a structure with filling factor $f = 0.74$ and a cover $n_c = 1.55$. (b) Minimum depth $\Gamma_{es,\min}$ of the evanescent wave inside the photonic crystals as a function of the cover refractive index, for the different filling factors considered in Fig. 4.

A final remark is in order: According to Fig. 5(a), favorable conditions to excite the Dyakonov surface waves in actual photonic metamaterials appear to occur near the cutoff condition for coupling with the extraordinary continuous spectrum. Importantly, as the plot also shows, under such conditions the penetration depth of the surface wave in the cover is the smallest; therefore, the spatial sensitivity of the waves to refractive-index surface perturbations is the highest. This immediately suggests the possibility to employ the corresponding waves to interrogate the surface properties with subwavelength resolution.

To conclude, we stress that the key concept put forward here is the exploitation of the anisotropy of photonic metamaterials to meet the special conditions at which Dyakonov surface waves exist. To posit the physical idea, here we focused in the long-wavelength, or homogenized limit, but the exploration of the existence of Dyakonov-type resonance phenomena for progressively shorter wavelengths is a fascinating question that immediately comes to mind and that needs to be addressed with more involved mathematical approaches. We finally notice that Dyakonov waves exist in more complex settings that those examined here, such as in ultrathin nanoscale waveguides or in nonlinear self-focusing materials [23], systems where the concept shown here is expected to hold too.

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