# Surface lattice solitons in diffusive nonlinear media

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We address the properties of surface solitons supported by optical lattices imprinted in photorefractive media with asymmetrical diffusion nonlinearity. Such solitons exist only in finite gaps of the lattice spectrum. In contrast to latticeless geometries, where surface waves exist only when nonlinearity deflects light toward the material surface, the surface lattice solitons exist in settings where diffusion would cause beam bending against the surface. © 2008 Optical Society of America

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Several optical materials intrinsically exhibit asymmetrical nonlinear responses that cause the development of strong shape asymmetries or even self-bending of light beams. Among such materials are photorefractive crystals [1–6] with a diffusion nonlinearity. The diffusion nonlinearity can cause substantial transverse beam displacements when acting in combination with drift nonlinearity in biased crystals. In the presence of interfaces diffusion nonlinearity results in the formation of localized surface states when bending toward the interface is compensated by total internal reflection [7–10]. Such surface waves are an example of nonlinear surface waves observed at the interface of natural uniform materials [11]. Combining diffusion and drift nonlinearity may also result in surface wave formation [12].

Surface waves can also be excited at moderate powers at the interface between a uniform medium and semi-infinite lattice [13,14]. They were found not only in focusing but also in defocusing media [15–18] and in both one- and two-dimensional [19,20] geometries. To date, surface lattice solitons were mostly studied in materials featuring a symmetrical local nonlinear response. The impact of the asymmetrical nonlocal nonlinear response on lattice solitons was studied only in infinite periodic structures [21,22]. In this Letter we address finite lattices and reveal that nonlocal asymmetrical diffusion nonlinearity can result in surface soliton formation in finite gaps in the lattice spectrum. In contrast to the case of interface of uniform diffusive medium, such solitons exist when diffusion nonlinearity would cause light bending not only toward the lattice edge but also when it bends light against it.

The propagation of the light beam along the $\xi$ axis in the vicinity of the interface of a semi-infinite optical lattice imprinted in an unbiased photorefractive crystal with intrinsic diffusion nonlinearity can be described by the nonlinear Schrödinger equation

$$i\frac{\partial q}{\partial \xi} = -\frac{1}{2}\frac{\partial^2 q}{\partial \eta^2} + \frac{\mu q}{1+S|q|^2}\frac{\partial |q|^2}{\partial \eta} - pR(\eta)q, \qquad (1)$$

where the transverse $\eta$ and longitudinal $\xi$ coordinates are scaled to the beam width $x_0$ and diffraction length $k_0x_0^2$, respectively; $q=k_0n_0(x_0k_bTr_e/2eI_d)^{1/2}A$ is the complex field amplitude; $k_b$ is the Boltzmann constant; $T$ is the temperature; $r_e$ is the electro-optic coefficient; $I_d$ is the dark irradiance; $e$ is the charge of free carriers; $n_0$ is the refractive index; $S=2e/k_0^2x_0n_0^2k_bTr_e$ is the saturation parameter; the parameter $p=k_0^2x_0^2\delta n/n_0$ is proportional to the depth of the refractive index modulation $\delta n$; and the lattice profile is described by the function $R(\eta)=0$ for $\eta<0$ and $R(\eta)=[1-\cos(\Omega\eta)]/2$ for $\eta\geqslant 0$. Such refractive index landscapes can be fabricated in suitable materials [14,17,18]. When $\mu=1$, a light beam launched inside the lattice tends to self-bend toward the bulk lattice, while when $\mu=-1$ light tends to self-bend toward the lattice edge. Here we set $\Omega=2$ and $S=0.5$. Equation (1) conserves the energy flow $U=\int_{-\infty}^{\infty}|q|^2 d\eta$.

We search for lattice surface soliton solutions in the form of $q(\eta,\xi)=w(\eta)\exp(ib\xi)$. Such states only exist for $b$ values falling into the gaps of the lattice spectrum [Fig. 1(a)]. The tails of the surface solitons decay in the uniform medium only when $b\geqslant 0$. These constraints approximately determine the domain of existence of surface solitons in diffusive medium. Surprisingly, such solitons can exist only in finite gaps of the lattice spectrum. Representative profiles of surface solitons belonging to the first gap are shown in Fig. 2.

In contrast to the case of uniform media, in the lattice surface solitons exist for both signs of $\mu$. The intensity maxima in the first lattice channel are shifted in opposite directions for opposite signs of $\mu$ [Fig. 2(a)]; moreover, when $\mu=1$ the soliton localization in-

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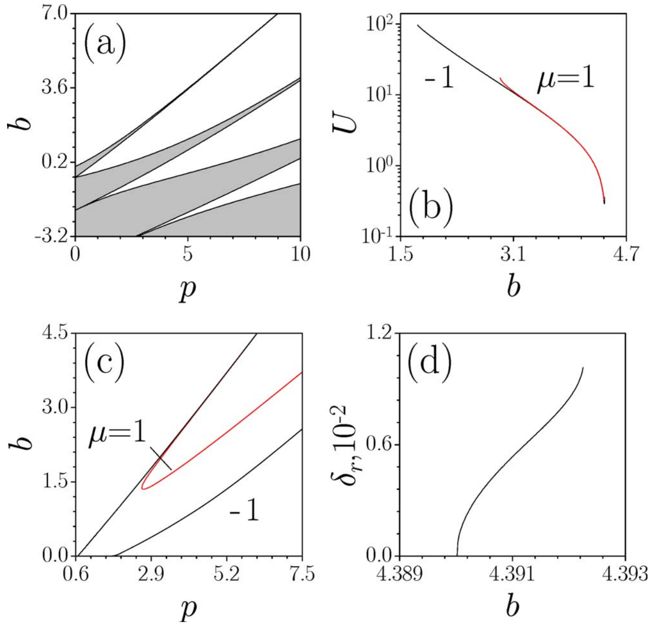


Fig. 1. (Color online) (a) Floquet–Bloch spectrum of infinite lattice. Gray regions show bands and white regions correspond to gaps. (b) Energy flow versus propagation constant for $\mu=1$ [gray curve (red online)] and $\mu=-1$ (black curve) at $p=6$. (c) Domain of existence of surface solitons on the $(p,b)$ plane for $\mu=1$ [gray curve (red online)] and $\mu=-1$ (black curve). (d) Real part of perturbation growth rate versus propagation constant at $p=6$, $\mu=1$. In all cases $\Omega=2$.

side the lattice is lower than when $\mu=-1$. This is because in the former case Bragg reflection from the periodic structure responsible for near-surface localization also counteracts the self-bending against the surface.

There exist both lower and upper cutoffs on $b$. When $\mu=-1$ the lower cutoff for solitons from the first gap coincides with the lower gap edge at $p \geqslant p_{\rm cr}^1$, while at $p_{\rm cr}^1 > p \geqslant p_{\rm cr}^2$ one has $b_{\rm low}=0$ [Fig. 1(c)]. In this case $p_{\rm cr}^1 \approx 1.78$ and $p_{\rm cr}^2 \approx 0.66$ correspond to the lattice depths at which the lower and upper gap edges cross the line $b=0$. The upper cutoff $b_{\rm upp}$ is a bit smaller but still very close to the upper gap edge. For $p > p_{\rm cr}^1$ at $b \to b_{\rm low}$ and at $b \to b_{\rm upp}$ surface solitons largely expand into the lattice region but are well localized in the uniform medium [Figs. 2(b) and 2(c)]. In contrast, at $p_{\rm cr}^1 > p \geqslant p_{\rm cr}^2$ for $b \to b_{\rm low}$ surface solitons feature an almost linearly decreasing tail inside the uniform medium but remain localized in the lattice [Fig. 2(d)]. The energy flow is a nonmonotonic function of $b$, it decreases with $b$ in most parts of the existence domain, but at $b \to b_{\rm low}$ the derivative $dU/db$ becomes positive so that surface solitons exist only above a certain energy flow threshold [Fig. 2(b)]. The existence of such a threshold is entirely a surface effect.

We found that at $\mu=1$ the domain of existence of surface solitons is much narrower than when $\mu=-1$ [Fig. 1(c)]. This is because the lattice cannot compensate the effects of self-bending for beams with sufficiently high amplitudes, and deeper lattices are necessary to support surface solitons with higher peak intensities. As a result, the domain of existence for surface solitons shrinks with a decrease in lattice depth and such solitons cease to exist for $p < p_{\rm cr}^3 \approx 2.61$, which exceeds $p_{\rm cr}^1, p_{\rm cr}^2$. The domain of existence for surface solitons for $\mu=1$ is located closer to the upper gap edge [Fig. 1(c)]. When $b \to b_{\rm upp}$ surface solitons strongly penetrate into the lattice [Fig. 2(e)], while at $b \to b_{\rm low}$ an increase of the energy flow is accompanied by a gradual equilibration of amplitudes of oscillations in the first and second lattice channels [Fig. 2(f)]. In this case the tangential line to the $U(b)$ curve becomes vertical in both cutoffs [Fig. 1(b)]. Surface lattice solitons for $\mu=1$ also exist only above a threshold energy flow. We also found surface solitons with similar properties in other finite gaps of the lattice spectrum for both signs of $\mu$.

A linear stability analysis for perturbed solutions $q=(w+u+iv)\exp(ib\xi)$ (here $u$ and $v$ are real and imaginary parts of perturbation that can grow with the complex rate $\delta=\delta_r+i\delta_i$ upon propagation) in Eq. (1) has shown that surface solitons in a diffusive medium are stable in the largest part of their existence domain for both signs of $\mu$. Only close to the upper

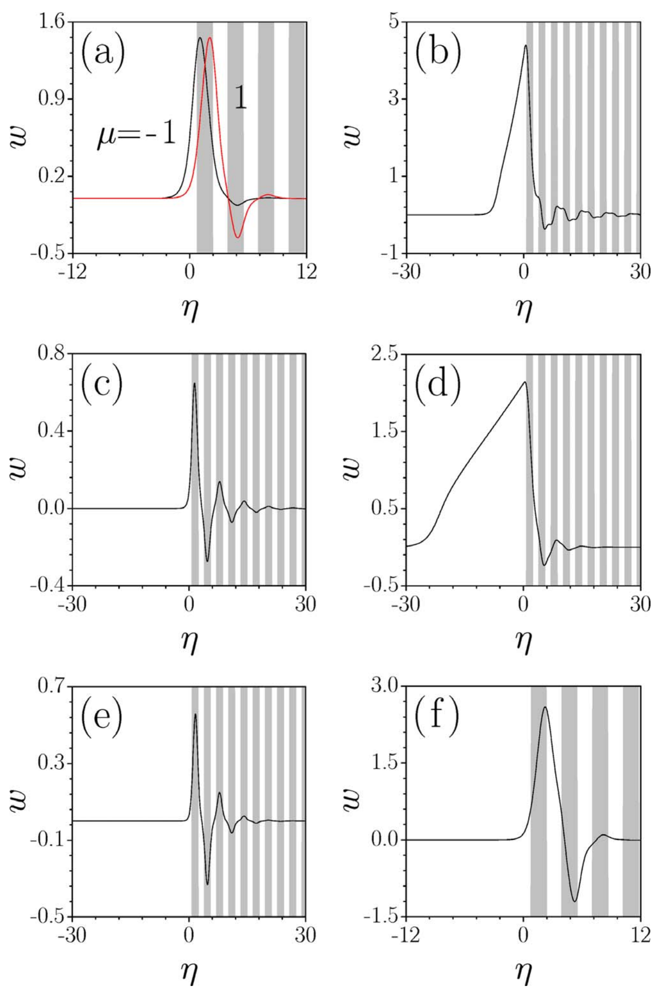


Fig. 2. (Color online) Profiles of surface solitons at (a) $b=1.9$, $p=3.5$, $\mu=-1$ (black curve) and $\mu=1$ [gray curve (red online)]; (b) $b=0.62$, $p=3.5$, and $\mu=-1$; (c) $b=2.27$, $p=3.5$, $\mu=-1$; (d) $b=0.1$, $p=1.5$, and $\mu=-1$; (e) $b=3.53$, $p=5$, and $\mu=1$; and (f) $b=2.4$, $p=5$, and $\mu=1$. In gray regions $R(\eta) \geqslant 1/2$, while in white regions $R(\eta) < 1/2$. In all cases $\Omega=2$.

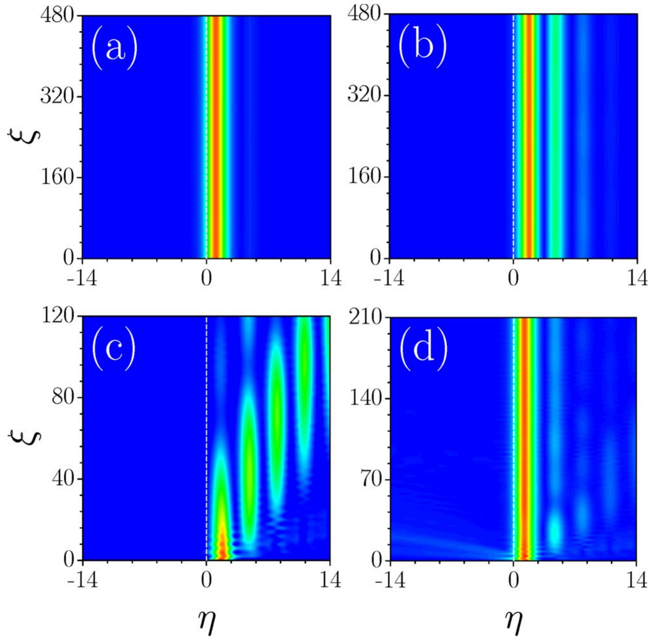


Fig. 3. (Color online) Stable propagation of perturbed surface solitons corresponding to (a) $b=1.9$, $p=3.5$, and $\mu=-1$, and (b) $b=2.23$, $p=3.5$, $\mu=1$. Broadband white noise was added into input field distributions. Dynamics of propagation of a Gaussian beam launched into the first channel of the lattice with $p=3$ at (c) $\mu=1$ and (d) $\mu=-1$. In all cases field modulus distributions are shown, white dashed lines indicate interface position, and lattice frequency $\Omega=2$.

cutoff one encounters a domain of exponential instability, which coincides with the region where $\mathrm{d}U/\mathrm{d}b \geqslant 0$ [see Fig. 1(d) for dependence $\delta_r(b)$ for $\mu=1$]. Such an instability domain exists for both signs of $\mu$. For $b$ values close to the lower gap edge surface solitons suffer from very weak oscillatory instabilities typical for gap solitons. Figures 3(a) and 3(b) show stable propagation of perturbed surface solitons at $\mu=\pm 1$.

Importantly, the excitation dynamics for surface solitons out of arbitrary inputs strongly differs for opposite signs of $\mu$, since for $\mu=-1$ the bending toward the lattice edge facilitates a surface wave formation, while for $\mu=1$ light tends to drift into the lattice bulk. Thus, higher energy flows are required for the excitation of surface solitons when $\mu=1$. This is illustrated in Figs. 3(c) and 3(d), where a Gaussian beam $q|_{\xi=0}=\exp[-(\eta-\pi/\Omega)^2]$, whose energy flow far exceeds the thresholds for the surface wave formation for both signs of $\mu$, excites the surface wave for $\mu=-1$, but diffracts when $\mu=1$. In the latter case the input energy flow has to be further increased to achieve a surface soliton formation. Surface solitons may be excited with input beams carrying much higher energies than the energy flows of surface solitons in the vicinity of lower cutoff. In this case, despite a strong radiation, a share of input energy flow usually remains trapped in the near-surface channel. For $p<p_{\mathrm{cr}}^2$ when $\mu=-1$ and for $p<p_{\mathrm{cr}}^3$ when $\mu=1$, surface soliton formation is not possible, irrespective of the shape and energy flow of the input beam.

Summarizing, we introduced gap surface solitons supported by asymmetrical nonlocal diffusion nonlinearity at the interface of optical lattices. Such solitons exist for both signs of the diffusive nonlinearity, in contrast to previous findings reported for latticeless geometries.

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