# Light bullets in optical tandems

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We address the concept of three-dimensional light bullet formation in structures where nonlinearity and dispersion are contributed by different materials, including metamaterials, which are used at their best to create suitable conditions where bullets can form. The particular geometry considered here consists of alternating rings made of highly dispersive but weakly nonlinear media and strongly nonlinear but weakly dispersive media. We show that light bullets form for a wide range of parameters. © 2009 Optical Society of America

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The formation of fully three-dimensional self-trapped wavepackets, or light bullets, is one of the most exciting, yet experimentally unsolved, problems in optics [1,2]. Light bullets are spatiotemporal solitons that form when a suitable nonlinearity arrests both spatial diffraction and temporal group-velocity dispersion. In principle, bullets may be supported by a variety of nonlinear mechanisms, even though potential solutions tend to be highly prone to dynamic instabilities. Their experimental realization faces two central challenges, namely, elucidation of a nonlinearity mechanism that supports light bullets as stable entities and realization of a physical setting where nonlinearity, diffraction, and dispersion are all present with suitable strength without introducing too high propagation losses. A variety of strategies have been found that solve the first challenge. These include quadratic nonlinear media that support solitons for all physical dimensions [3] and where two-dimensional bullet formation was achieved by generating dispersion via achromatic phase matching at the expense of one spatial dimension [4,5]; saturable [6] and nonlocal [7,8] media; materials with competing nonlinearities [9,10]; settings where higher-order effects such as fourth-order dispersion may play a stabilizing role [11]; propagation in optical lattices [12–14]; and filamentation [15,16] just to name a few. However, to date, the second challenge remains unsolved.

A potential approach to overcome this limitation is based on the concept of engineered structures composed of different materials featuring either strong nonlinearity or strong suitable group-velocity dispersion but not necessarily both together at a given wavelength. Thus, each material is to be used at its best for the purpose at hand. Implementation of such strategy along the longitudinal direction showed that light bullet formation is possible for significantly large tandem domains in the case of quadratic solitons [17]. In this Letter we put forward the concept that stable bullets do form in transverse radially periodic structures consisting of alternating rings made of a highly dispersive linear material and rings made of a strongly nonlinear material. We find that bullet stability depends crucially on whether the central domain is linear or nonlinear. Here we address materials with cubic saturable nonlinearity, but the concept is expected to hold for different nonlinearities.

We address the propagation of a light beam along the $\xi$ axis of a radial tandem described by the nonlinear Schrödinger equation for dimensionless field amplitude $q(\eta,\zeta,\tau,\xi)$:

$$i\frac{\partial q}{\partial \xi}=-\frac{1}{2}\left(\frac{\partial^2 q}{\partial \eta^2}+\frac{\partial^2 q}{\partial \zeta^2}\right)+\frac{\beta(\eta,\zeta)}{2}\frac{\partial^2 q}{\partial \tau^2}+\sigma(\eta,\zeta)\frac{q|q|^2}{1+S|q|^2}. \quad (1)$$

Here $\xi=z/L_{\mathrm{difr}}$ is the propagation distance normalized to the diffraction length $L_{\mathrm{difr}}=k_0r_0^2$, $\eta=x/r_0$ and $\zeta=y/r_0$ are transverse coordinates normalized to the characteristic scale $r_0$, $\tau=t/t_0$ is the normalized (retarded) time, $\beta=L_{\mathrm{difr}}/L_{\mathrm{disp}}$, the dispersion length in each domain is defined as $L_{\mathrm{disp}}=|\partial^2k_0/\partial\omega^2|^{-1}t_0^2$, the quantity $\sigma(\eta,\zeta)=-1$ stands for focusing nonlinearity, while $\sigma(\eta,\zeta)=0$ stands for a linear medium, and $S$ is a saturation parameter. We assume a radially symmetric structure composed of periodically alternating rings exhibiting anomalous dispersion and weak nonlinearity, where $\beta=-2$ and $\sigma=0$, and weakly dispersive but highly nonlinear domains, where $\beta=-0.1$ and $\sigma=-1$. The radial width of the domains is $d$. Importantly, two types of geometries are possible, when the central domain exhibits nonlinearity and when it does not. The refractive index is set to be similar in all domains; thus the structure may be viewed as a *nonlinear lattice*. Note that two-dimensional solitons in geometries where nonlinearity and dispersion are transversally modulated at equal points have been shown to exist [18,19]. The essential ingredient of the approach put forward here, where nonlinearity and dispersion are strong *at different points* of the structure, should be properly appreciated.

Figure 1 illustrates the linear patterns obtained for different radial periods of the tandem in the absence of nonlinearity for Gaussian inputs, i.e., $q|_{\xi=0}=A\exp(-\eta^2-\zeta^2-\tau^2)$. For large domain widths diffraction resembles that in uniform media. When $\beta=-2$ in the central domain, dispersion is stronger than diffraction and the intensity isosurfaces have ellipsoidal shapes elongated in time [Fig. 1(a)], while when $\beta=$

 

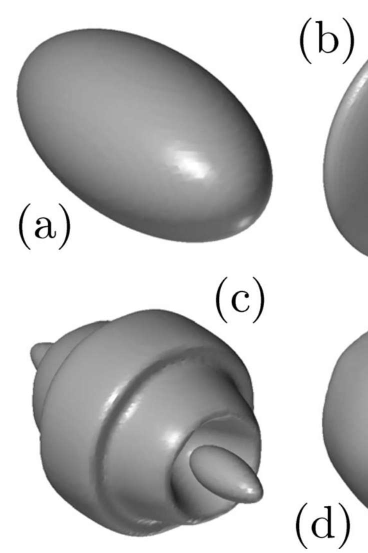


Fig. 1. Isosurface plots showing output patterns after propagation at $\xi=2$ of input Gaussian signals in linear tandem structures with (a), (b) $d=20$; (c) $d=0.8$; and (d) $d=0.4$. In (a) the dispersion in the central domain is set to $\beta=-2$. In (b)–(d), $\beta=-0.1$.

$-0.1$ one gets Fig. 1(b). Decreasing the domain width results in distortions of the input, since it covers several domains with substantially different dispersions [Fig. 1(c)]. When the domain width is sufficiently small, the beam experiences the average dispersion of the structure. The corresponding intensity isosurfaces approach spheres [Fig. 1(d)]. Thus, addition of nonlinearity may result in the formation of light bullets by compensating diffraction and the effective dispersion.

We search for light bullet solutions in the form $q=w(r,\tau)\exp(ib\xi)$, where $b$ is the propagation constant. Solutions approach those of uniform media with average dispersion and nonlinearity only at the limit $d\rightarrow 0$, but in general they correspond to the exact nonlinear radial lattice defined by Eq. (1). Here we assume nonlinearity saturation to avoid collapse that occurs in Kerr media. To conduct stability analysis, the perturbed solutions of Eq. (1) can be written as $q=[w+u\exp(ik\phi)+v^*\exp(-ik\phi)]\exp(ib\xi)$, where $u,v$ are small perturbations that can grow with rate $\delta$ upon propagation and $k$ is an azimuthal perturbation index. Typical shapes of light bullets are shown in Fig. 2. For suitable parameters, bullets may cover several radial domains and they may feature pronounced shape modulations. The intensity distribution features a ringlike shape in the $(\eta,\zeta)$ plane, an effect that is most pronounced for tandems with a nonlinear central domain. Bullets expand substantially at low and high amplitudes, the latter being a consequence of the nonlinearity saturation. At certain $b$ they acquire minimal width that increases with $S$. The total energy carried by the bullets $U=2\pi\int_0^\infty r dr\int_{-\infty}^{\infty} w^2 d\tau$ is a nonmonotonic function of $b$ [Fig. 3(a)]—it diverges in an upper cutoff $b_{\text{upp}}$ that grows with decreasing $S$ and that only slightly depends on $d$. When $b$ decreases, the bullet energy reaches a minimal value close to a lower cutoff and

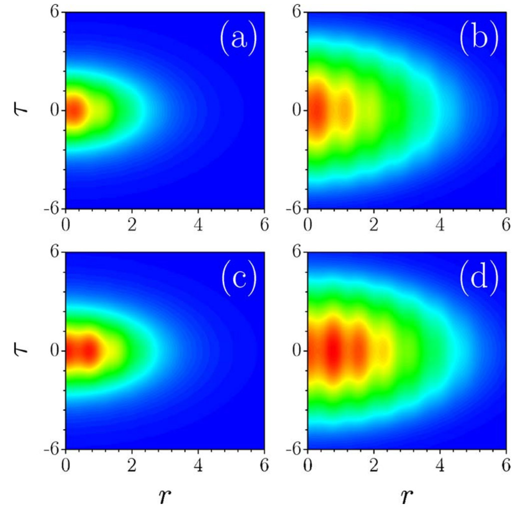


Fig. 2. (Color online) Soliton profiles in tandem structures with $d=0.4$ and $S=0.5$. (a), (c) $b=0.3$; (b), (d) $b=0.85$. In (a), (b) the central domain is linear, while in (c), (d) the central domain is nonlinear.

then starts increasing as $b\rightarrow 0$. Thus, in the nonlinear lattice defined by the radial tandems light bullets always exist above a threshold energy that diminishes with increasing the domain width $d$ and with decreasing the saturation parameter $S$. Figure 3(b) illustrates how the energy carried by the bullets evolves with decreasing domain width $d$. The difference between the corresponding $U(b)$ curves diminishes for any $b$ value (and not only at $b\rightarrow 0$ when bullets are always broad and cover many domains) as the domain width $d$ becomes smaller. This is consistent with the expectation that light evolution in the

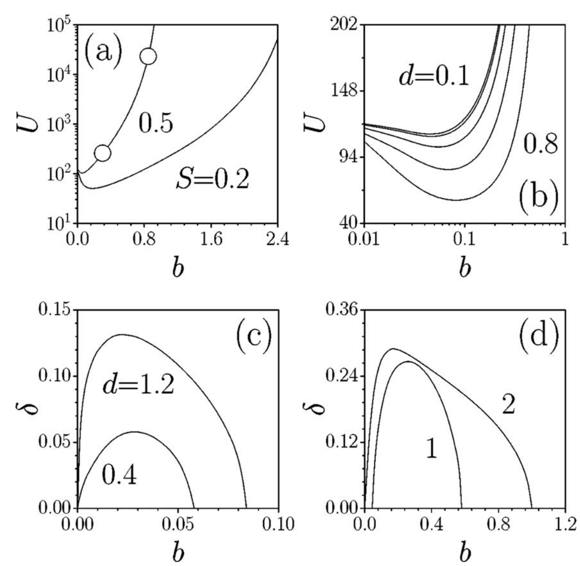


Fig. 3. Energy versus propagation constant for (a) different $S$ values at $d=0.4$ and (b) different $d$ values at $S=0.5$. The circles in (a) correspond to the solitons shown in Figs. 2(a) and 2(b). In (b) the domain width takes values $d=0.8$, 0.6, 0.4, 0.2, and 0.1 from the lower to the upper curve. (c) Perturbation growth rate versus $b$ at $k=0$, $S=0.5$. In (a)–(c) the central domain is linear. (d) Growth rate versus $b$ at $k=1$, $d=1.2$, $S=0.5$ in the structure with linear (1) and nonlinear (2) central domains.

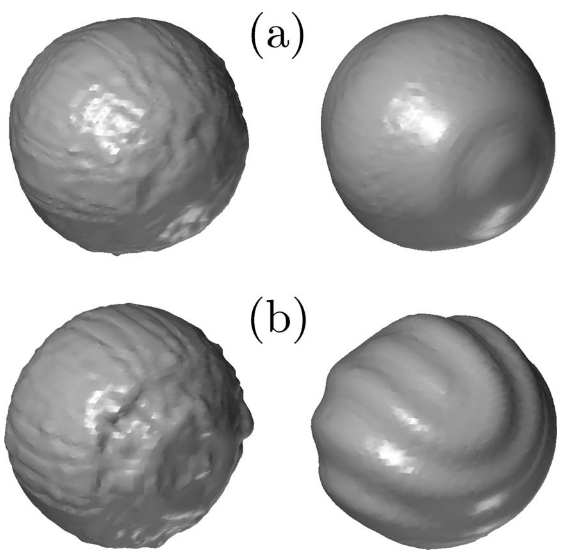


Fig. 4. (a) Isosurface plots at $\xi=0$ (left) and $\xi=128$ (right) showing stable propagation of a light bullet in a tandem structure with a linear central domain with $\beta=-2$, and (b) isosurface plots at $\xi=0$ (left) and $\xi=72$ (right) showing unstable propagation of a light bullet in tandem with a nonlinear central domain with $\beta=-0.1$. In all cases $b=0.3$, $d=0.4$, $S=0.5$.

structure with sufficiently small domains mimics the evolution in uniform media with average parameters. However, note that the central result of this Letter is that fully three-dimensional light bullets do exist far from those averaged conditions.

Linear stability analysis predicts that there exist two types of perturbations that may destabilize light bullets in the tandems. The first type of instability corresponds to the azimuthal perturbation index $k=0$, and it is possible only for $b\to 0$ in the narrow region where the dependence $U(b)$ exhibits a negative slope $dU/db\leq 0$. Such an instability domain quickly shrinks, and the growth rate (which is purely real) diminishes when $d$ decreases [Fig. 3(c)]. Azimuthal instabilities associated with $k=1$ may also appear when solutions acquire a ringlike shape. Such instability is absent in tandems with a linear central domain (i.e., in such tandems bullets are stable almost in the entire existence domain) unless $d$ notably exceeds 1. In contrast, in tandems with a nonlinear central domain the azimuthal instability is much stronger, so that it may result in the destabilization of the bullets in almost their entire existence domain. Such strong instability may be present even for domain widths $d\sim 0.2$. Examples of instabilities are shown in Fig. 3(d), where the curves $\delta(b)$ for $k=1$ are compared for bullets supported by tandems with linear and nonlinear central domains with $d=1.2$. While stabilization in tandems with a nonlinear central domain is possible, the stability region in terms of $b$ is narrow, and stabilization occurs at much higher powers than in tandems with a linear central domain. Therefore, instability suppression for bullets supported by tandems with a nonlinear central core requires substantially smaller domain widths and higher nonlinearity saturation. These predictions are in agreement with full numerical propagation of perturbed solutions. Thus, Fig. 4(a) illustrates how a typical bullet supported by a tandem with a linear central domain cleans up itself and keeps its shape upon propagation in a robust way, while a similar stationary solution supported by a tandem with a nonlinear central domain that eventually decays is shown in Fig. 4(b). Note that in the latter case the input signal goes through significant oscillations and shifts (not directly visible in the isosurface plots).

In summary, stable light bullets can form in radial tandem structures made of alternating linear and nonlinear domains with drastically different dispersion and nonlinear properties. The setting addressed is a particular example, the important result being that light bullets form in structures where materials are used at their best to meet the requirements needed to support light bullets in practice. It is our belief that these findings motivate a program devoted to building suitable metamaterial structures.

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